J. Fluid Mech. (1971), *vol.* 47, *part* 4, *pp.* 827–828
Printed in Great Britain

REVIEWS

Ellipsoidal Figures of Equilibrium. By S. CHANDRASEKHAR. Yale University Press, 1969. 252 pp. $10 or £4.50.

The first chapter of this book is a historical introduction. The theory of gravitational equilibria of homogeneous uniformly rotating masses began in Newton's *Principia* (the oblateness of the rotating earth) and developed through the work of Maclaurin (the Maclaurin spheroids), Jacobi (the Jacobi ellipsoids), Meyer and Liouville (the relationship between the Maclaurin and Jacobi sequences), and Dirichlet, Dedekind and Riemann (the general problem of equilibrium of an ellipsoidal body, including internal motions with uniform vorticity).

Many of the great men of mathematics have obviously been attracted by the subject. Riemann, discussing a fundamental paper by Dirichlet, wrote: "The further development of his beautiful discovery has a particular interest to the mathematician even apart from its relevance to the forms of heavenly bodies which initially instigated these investigations". At this stage (1860), many aspects of the general theory of equilibria were unclear (for example, the relation of Riemann's ellipsoids to the Maclaurin spheroids). These aspects were to remain unclear for more than a hundred years, partly because of Poincaré's spectacular discovery in 1885 of a new sequence of pear-shaped configurations branching off the Jacobi ellipsoid sequence. Poincaré conjectured that the bifurcation of such pear-shaped configurations would lead "onwards stably and continuously to a planet attended by a satellite...". Also, Darwin conjectured that double stars may form in this way, and most subsequent effort (until 1924, when work on this subject apparently ceased) was side-tracked to follow these conjectures. The orderly development of the general theory of equilibria and stability remained far from complete, until Chandrasekhar reopened the subject in about 1960. The main purpose of this book is "towards bringing to the subject order and coherence in a common outlook": much of this order and coherence comes from a series of forty papers published mostly in *The Astrophysical Journal* by Chandrasekhar and his associates.

The second chapter develops the virial equations of various orders. These equations are the moments of the hydrodynamical equations. They have wider application in fluid dynamics and hydromagnetics, but their particular use here is that they allow exact solutions to these historical problems of equilibrium and stability to be obtained in an elementary way (as proved in chapter 4). The third chapter is a useful and complete account of the potentials of homogeneous and heterogeneous ellipsoids.

The Maclaurin spheroids are then discussed in detail. These are the simplest of the uniformly rotating homogeneous ellipsoidal figures, and define a one-parameter sequence. The book describes their various modes of oscillation, their stability, and the bifurcation of the Maclaurin sequence where the Jacobi ellipsoid sequence branches off. It is also shown that each point along the stable part of the Maclaurin sequence is a point of bifurcation in four different ways: the four types of figures branching off are the more general Riemann ellipsoids, which have internal motions of uniform vorticity.

In the same spirit Chandrasekhar then discusses the triaxial Jacobi and Dedekind ellipsoids and then the Riemann ellipsoids. This topic stems from Riemann's paper of 1860, which "contains some very surprising lapses and some definitely erroneous conclusions". In view of Riemann's unique place in science, there is included in the bibliographic notes to chapter 7 an interesting critical appraisal of Riemann's paper.

At present it seems that homogeneous gravitating equilibria are rarely observed in the universe, so their theory probably has little direct "relevance to the forms of heavenly bodies which initially instigated these investigations". Indirectly the subject does give dynamical astronomers a useful feel for sequences of equilibria and their relationships, including their oscillations and dynamical and secular stability. However, I believe that the subject's chief importance is its intrinsic interest, amply demonstrated by the attentions of great mathematicians over three hundred years.

This book is clearly written and certainly achieves its purpose of bringing order and coherence to the theory of ellipsoidal figures of equilibrium.

KENNETH FREEMAN

Turbomachines Hydrauliques et Thermiques. By MARCEL SÉDILLE. Masson et Cie. Vol. 3, 1969, 288 pp., 58 F. Vol. 4, 1970, 300 pp. 65 F.

Volume 1 of this series entitled *Incompressible Fluid Mechanics*, and volume 2, *Centrifugal and Axial Pumps*, have both been reviewed in the *Journal* (vol. 32, p. 825 and vol. 34, p. 822). The third volume entitled *Technical Thermodynamics* and the fourth, *Compressible Fluid Mechanics*, have now appeared to complete a notable set of engineering text-books. The author is a well-known engineer and also a distinguished teacher, and together with three collaborators has produced a most excellent account of thermo-fluid dynamics applied to machines. The approach is remarkable for the detailed description of fundamental theory presented with the utmost clarity and never failing to recall the engineering significance of what is discussed. Orders of magnitude and the practical importance of the physics are constantly being stressed. For instance the Clapeyron PV diagram for solid, liquid, and vapour phases is used to illustrate how full liquefied-gas bottles should be filled for safety. The many graphical illustrations and charts are first class and one is constantly aware of the authority of the author on both fundamental physics and engineering applications.

The third volume covers the laws of thermodynamics, the behaviour of mixtures of perfect gases, combustion, thermal and energy efficiencies. Chapter 7, on free enthalpy of mixtures, Gibbs's phase rule, liquefaction of gases and multiphase systems, is particularly good and also chapter 8 on the thermodynamics of bubbles and droplets where surface tension is important. Chapter 8 deals specifically with the use of thermodynamic diagrams, but throughout the graphical illustrations are used most intelligently.

Boiling and cavitation are described in detail, and although perhaps the description of the former is conventional the analysis of cavitation (also covered in volume 2) is unusually good.

To appreciate and understand the fourth volume it is necessary to have studied volume three. Because of the shortage of text-books in French on compressible flow this volume is intended as a definitive work on the subject and has drawn widely from international sources. It therefore goes further than is necessary for the study of thermal machines but produces a general picture for engineers of two-dimensional compressible flow. The eight chapters cover the usual topics of transonic and supersonic flow. Chapters 6, 7 and 8 are less conventional and provide a good account of compressible boundary layers in nozzles with heat exchange, the flow of real gases and of liquids and vapours, and unsteady flow in ducts. This again is a particularly comprehensive work on the subject and leaves out no essential steps for the student. In fact, given the four volumes, the student would be well prepared not only for his university course but for his future career.

The unity of the presentation, the consistent notation and the clarity of presentation would not have been possible without very close teamwork between the author and his assistants. It is doubtful whether any other author has covered such a wide field of thermo-fluids and machines so well. A truly remarkable effort and strongly recommended for those who can cope with very clearly written French and who are either engineers interested in fundamentals or advanced university students.

S. P. Hutton